

Problem 1.7

[Difficulty: 3]

1.7 Very small particles moving in fluids are known to experience a drag force proportional to speed. Consider a particle of net weight W dropped in a fluid. The particle experiences a drag force, $F_D = kV$, where V is the particle speed. Determine the time required for the particle to accelerate from rest to 95 percent of its terminal speed, V_t , in terms of k , W , and g .

Given: Small particle accelerating from rest in a fluid. Net weight is W , resisting force $F_D = kV$, where V is speed.

Find: Time required to reach 95 percent of terminal speed, V_t .

Solution: Consider the particle to be a system. Apply Newton's second law.

Basic equation: $\sum F_y = ma_y$

Assumptions:

1. W is net weight
2. Resisting force acts opposite to V

Then

$$\sum F_y = W - kV = ma_y = m \frac{dV}{dt} = \frac{W}{g} \frac{dV}{dt}$$

or

$$\frac{dV}{dt} = g \left(1 - \frac{k}{W} V \right)$$

Separating variables,

$$\frac{dV}{1 - \frac{k}{W} V} = g dt$$

Integrating, noting that velocity is zero initially,

$$\int_0^V \frac{dV}{1 - \frac{k}{W} V} = -\frac{W}{k} \ln \left(1 - \frac{k}{W} V \right) = \int_0^t g dt = gt$$

or

$$1 - \frac{k}{W} V = e^{-\frac{kgt}{W}}; \quad V = \frac{W}{k} \left(1 - e^{-\frac{kgt}{W}} \right)$$

But $V \rightarrow V_t$ as $t \rightarrow \infty$, so $V_t = \frac{W}{k}$. Therefore

$$\frac{V}{V_t} = 1 - e^{-\frac{kgt}{W}}$$

When $\frac{V}{V_t} = 0.95$, then $e^{-\frac{kgt}{W}} = 0.05$ and $\frac{kgt}{W} = 3$. Thus $t = 3W/gk$

